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Near-field effect in two-atom system

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Abstract. The self-consistent problem is solved for the interaction of two dipole atoms situated at arbitrary distance from one another with the field of quasiresonant light wave. Atoms are considered to be linear Lorenz oscillators. Polarizing fields inside the system include both Coulomb and retarding parts. The solutions obtained are investigated for the case when atoms have the same polarizabilities and interatomic distance is much less than external light wavelength. Formulas for electric fields inside and outside of small object are obtained. It is shown that longitudinal and transverse optical oscillations are possible to exist inside small two-atom object. Dispersion laws of these oscillations depend upon interatomic distance and upon angle between axis of the system and the direction of propagation of external wave. The field outside the small object in wave zone is linearly polarized with the choice of linear polarization of external field. However, the directions of polarization of these waves are different and depend essentially upon frequency. The amplitude of field outside small object in wave zone is shown to depend essentially on the frequency of external field and interatomic distance. The results obtained are treated as near-field effect in the optics of small objects making it possible to investigate the structure of small objects with optical radiation.

PACS. 42.25.Bs Wave propagation, transmission and absorption – 42.25.Gy Edge and boundary effects; reflection and refraction

1 Introduction

Optics of dielectrics is based on the fundamental concept about continuous matter where the spatial scale of medium, interatomic distance, is significantly less than wavelength of light. As a consequence of this concept the Fresnel laws of reflection and refraction take place in which the reaction of medium is concentrated at mathematical surface [1]. Fresnel coefficients of light reflection and transmission are used, in particular, for the description of photonic band gap in optical lattices where planes are separated by distance equal to half of wavelength [2].

However, there exist a lot of physical situations where near-field effect plays a significant role [3–7]. Theoretical consideration of such situations is based on the concept about discrete-continuous medium. According to this concept, any point of observation inside medium may be surrounded by Lorentz sphere. Dipoles within this sphere are discretely distributed. The field of dipoles in the center of Lorentz sphere is formed by Coulomb terms proportional to $1/R_a^3$ and retarding terms proportional to $1/R_a^2$ and $1/R_a$ where R_a is the distance between dipole a inside Lorentz sphere and its center. It was demonstrated in [4], that the field of dipoles inside Lorentz sphere is always different from zero because of field of $1/R_a$ type. Thus, optics of dielectrics has to be based on the concept of discrete-continuous medium where the idea of continuity is justified only outside Lorentz sphere. Lorentz sphere is closed for observation points inside medium and the role of discretely distributed atoms (structure factor) is insignificant. For instance, the role of near-field effect in refractive index of dielectrics with various types of symmetry comes to some percents. The situation changes drastically if the observation point is situated near the surface of medium. In that case Lorentz sphere is truncated and Coulomb field of $1/R_a^3$ type plays leading part in structure factor. Various physical situations were examined in [3–7] where near-field effect manifests itself under the influence of structure factor at different types of symmetry of discretely distributed atoms. In [3], the boundary-value problem of quantum optics was examined where excited atom spontaneously emitted a photon near the semi-infinite dielectric. It was shown that atomic lifetime changes significantly if the atom is in near zone relative to dielectric surface. In [4], the boundary-value problem of classical optics was considered where plane light wave was reflected and refracted on the flat surface of semi-infinite dielectric. It was shown that the non-Fresnel laws of light reflection and refraction appear for observation points in near zone. It would be interesting to examine such a physical situation when near-field effect manifests itself in observation points in wave zone relative to the surface. With that end in view the boundary-value problem of classical optics was examined where plane light wave interacts with

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ultra-thin nonabsorbing dielectric film consisting of several atomic layers [5]. The processes of reflection and transmission of light in view of near-field effect were considered and comparative calculation of reflectance and transmittance was performed on the basis of Eiry formula which use the idea of continuous film. Near-field effect was shown to be significant in observation points at arbitrary distance from the surface of film. In [6,7], the boundary-value problem of nonlinear resonant optics of super thin films was solved in view of near-field effect. The considerable role of near-field effect was shown in phenomena of optical bistability, dynamic shift of resonance etc.

The purpose of this work is to investigate the optical properties of the system of two dipole atoms situated at the arbitrary distance from one another. The optical properties of the system are investigated with the probe light field represented as a plane wave. We will demonstrate that mutual influence of atoms in near zone may be investigated in observation points located in wave zone relative to the system of atoms. The various manifestations of this mutual influence will be treated as near-field effect in small optical object.

The significant number of theoretical and experimental works [1,8] is devoted to the optics of small spherical particles. However, theoretical consideration of these objects was based on the concept about continuous mediums with a certain dielectric permeability independent on coordinates. This work allows us to investigate fields inside and outside of small objects with various distributions of atoms inside these objects. It will be shown that the presence of another atom essentially changes the amplitude-phase distribution of light field in wave zone in comparison with distribution of field of individual dipole [1]. This and other optical properties of small object formed by two dipole atoms we also call a near-field effect. The theoretical consideration of this effect will be based on the solution of self-consistent set of equations for electric field strength and atomic variables.

The large number of works is devoted to the theoretical research of the problem of two atoms in the field of radiation. Actually, the development of the theory of optical superradiation [9] and the derivation of integral equations of electromagnetic waves propagation in a medium [10] start with the solution of this problem. Unlike [9] where the collective spontaneous radiation of two atoms and the kinetics of this process were considered, the problem of two classical oscillators in classical field of light wave is examined in this paper. Thus, the basic attention is given to the investigation of spatial distribution of field inside and outside of small object in near and wave zones. The key purpose of this paper is the theoretical research of the capability of optical field application for the investigation of small objects with sizes much less than wavelength.

2 Master equations

We define the microscopic field of light wave $\mathbf{E}(\mathbf{r},t)$ in some observation point $\mathbf{r}$ in an instant t with the following equation [1]:

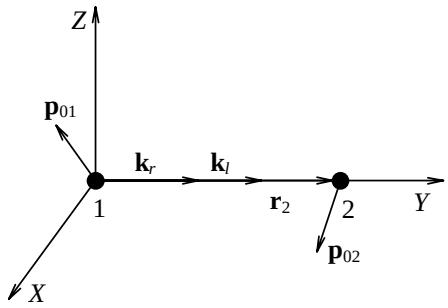

Fig. 1. The scheme of arrangement of two dipole atoms interacting with the field of light wave.

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_I(\mathbf{r},t)+\sum_{j=1}^{2}\mathrm{rot}\,\mathrm{rot}\frac{\mathbf{p}_j(t-R_j/c)}{R_j}, \qquad (2.1)$$

where $\mathbf{E}_I(\mathbf{r},t)$ is the electric field strength of external light wave propagating with speed of light c, $\mathbf{p}_j$ is the induced dipole moment of atom j which is considered to be the linear function of field $\mathbf{E}(\mathbf{r}_j,t-R_j/c)$ in the location of atom j; $R_j=|\mathbf{r}-\mathbf{r}_j|$ where $\mathbf{r}_j$ is the radius-vector of atom j relative to the origin of coordinates located in the center of one of atoms, for example, atom 1 (Fig. 1). Differentiation in (2.1) is performed with respect to the coordinates of observation point. In the special case when the observation point is in the location of one of atoms, we obtain from (2.1) the system of two equations for unknown quantities $\mathbf{E}(\mathbf{r}_1,t)$ and $\mathbf{E}(\mathbf{r}_2,t)$. Having defined these quantities, it is possible to find a field in other observation points with the aid of equation (2.1).

Equation (2.1) must be supplemented by equations for atomic variables. Let atoms be a Lorenz oscillators [11]. In this case the vector of induced dipole moment $\mathbf{p}_j$ takes the following form:

$$\mathbf{p}_j=e(\mathbf{u}_j-\mathrm{i}\mathbf{v}_j)\exp(-\mathrm{i}\omega t)+\mathrm{cc},\quad j=1,2, \qquad (2.2)$$

where e is the charge of electron, ω is the frequency of oscillations. Quantities $\mathbf{u}_j$ and $\mathbf{v}_j$ depend on the atomic position and on time as the atomic eigenfrequencies ω_1 and ω_2 differ from the frequency ω of external light wave. However, $\mathbf{u}_j$ and $\mathbf{v}_j$ will vary slowly in time if the differences $\omega-\omega_1$ and $\omega-\omega_2$ are small. In this case the following inequalities are held:

$$\begin{aligned}&|\dot{\mathbf{u}}_j|\ll\omega\,|\mathbf{u}_j|\,,\quad |\ddot{\mathbf{u}}_j|\ll\omega^2\,|\mathbf{u}_j|\,,\\ &|\dot{\mathbf{v}}_j|\ll\omega\,|\mathbf{v}_j|\,,\quad |\dot{\mathbf{v}}_j|\ll\omega^2\,|\mathbf{v}_j|\,.\end{aligned} \qquad (2.3)$$

These conditions allow us to rewrite an equation of motion for dipole j

$$\ddot{\mathbf{p}}_j+\frac{2}{\tau_0}\dot{\mathbf{p}}_j+\omega_j^2\mathbf{p}_j=\frac{e^2}{m}\mathbf{E}(\mathbf{r}_j,t) \qquad (2.4)$$

as

$$\frac{\partial}{\partial t}(\mathbf{u}_j-\mathrm{i}\mathbf{v}_j)=\left(-\mathrm{i}\Delta_j-\frac{1}{T}\right)(\mathbf{u}_j-\mathrm{i}\mathbf{v}_j)+\mathrm{i}\kappa_0\mathbf{E}_{0j}, \qquad (2.5)$$

where m is the mass of electron, $1/\tau_0$ is the relative rate of energy decay of isolated dipole [11], $\kappa_0 = e/\mathrm{m}\omega$; T^{-1} is the total rate of decay of oscillator which can differ from the rate of decay of isolated oscillator, $\Delta_j = \omega_j - \omega$ is the detuning from resonance. We define $\mathbf{E}_{0j}$ from the expression for field

$$\mathbf{E}(\mathbf{r}_j, t) = \mathbf{E}_0(\mathbf{r}_j) \exp(-\mathrm{i}\omega t) + \mathrm{cc} \tag{2.6}$$

as $\mathbf{E}_{0j} = \mathbf{E}_0(\mathbf{r}_j)$.

Equations (2.5, 2.1) form the closed set of equations where the reciprocal influence of field and atoms is taken into account in a self-consistent way. Thus [1],

$$\mathrm{rot}\,\mathrm{rot}\frac{\mathbf{p}_j}{R_j} \equiv \frac{3\left([\mathbf{p}_j]\,\mathbf{n}_j\right)\mathbf{n}_j - [\mathbf{p}_j]}{R_j^3} + \frac{3\left([\dot{\mathbf{p}}_j]\,\mathbf{n}_j\right)\mathbf{n}_j - [\dot{\mathbf{p}}_j]}{cR_j^2} + \frac{\left([\ddot{\mathbf{p}}_j]\,\mathbf{n}_j\right)\mathbf{n}_j - [\ddot{\mathbf{p}}_j]}{c^2 R_j}, \tag{2.7}$$

where symbol $[\ldots]$ means that the appropriate quantity is determined in an instant $t - R_j/c$, $\mathbf{n}_j = \mathbf{R}_j/R_j$. In the special case when the observation point coincides the site of one of atoms, R_j represents the interatomic distance R. First addend in (2.7) corresponds to the Coulomb field of dipole and other addends correspond to the retarding field of dipole in observation point $\mathbf{r}$. Field (2.7) is the polarizing field of dipole j. This field differs from scattered field [12]. Further on the spatial distribution of Coulomb and retarding polarizing fields in various observation points will be investigated in view of self-consistent interaction of two dipoles using the steady solution of equation (2.5).

3 Electric field of light wave inside small object

Placing the origin of coordinates in point $\mathbf{r}_1$, we have: $\mathbf{r}_1(0,0,0)$ and $\mathbf{r}_2(0,R,0)$ (Fig. 1). Choosing coordinate system in this way, we obtain the following set of equations for unknown values of fields $\mathbf{E}(\mathbf{r}_1, t)$ and $\mathbf{E}(\mathbf{r}_2, t)$ in the location of each atom:

$$\mathbf{E}(\mathbf{r}_i, t) = \mathbf{E}_I(\mathbf{r}_i, t) + \frac{3\left[p_j^y\right]\mathbf{y}_0 - [\mathbf{p}_j]}{R^3} + \frac{3\left[\dot{p}_j^y\right]\mathbf{y}_0 - [\dot{\mathbf{p}}_j]}{cR^2} + \frac{\left[\ddot{p}_j^y\right]\mathbf{y}_0 - [\ddot{\mathbf{p}}_j]}{c^2 R} \qquad (i \neq j), \tag{3.1}$$

where $\mathbf{y}_0$ is the unit vector in the direction of y-axis.

Let the external field be of the following form:

$$\mathbf{E}_I(\mathbf{r}_i, t) = \mathbf{E}_{0I} \exp[\mathrm{i}(\mathbf{k}_0\mathbf{r}_i - \omega t)] + \mathrm{cc}, \tag{3.2}$$

where $\mathbf{E}_{0I}$ is the constant amplitude, k_0 is the wave vector with modulus equal to $k_0 = \omega/c$. Induced dipole moments and field in points $\mathbf{r}_1$ and $\mathbf{r}_2$ are determined by expressions (2.2, 2.6) where $\mathbf{p}_{0j} = e\,(\mathbf{u}_j - \mathrm{i}\mathbf{v}_j)$ and $\mathbf{E}_{0j}$ are complex.

Substituting equations (3.2, 2.2, 2.6) into equation (2.1) and singling out uniformly oscillating factors, we obtain for the case of steady solution of equation (2.5) the following equality:

$$\mathbf{p}_{0j} = \alpha_j \mathbf{E}_{0j}, \tag{3.3}$$

where

$$\alpha_j = \frac{e^2}{m}\frac{1}{\omega_j^2 - \omega^2 - 2\mathrm{i}\omega/T} \tag{3.4}$$

is the polarizability of atom j [11].

Substituting (3.3) into the set of equations (3.1), after appropriate transformations we obtain the following related equations:

$$\begin{aligned} p_{01}^y &= \alpha_1\left\{E_{0I}^y + 2Gp_{02}^y \exp(\mathrm{i}k_0 R)\right\}, \\ p_{02}^y &= \alpha_2\left\{E_{0I}^y \exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{R}) + 2Gp_{01}^y \exp(\mathrm{i}k_0 R)\right\}, \end{aligned} \tag{3.5}$$

$$\begin{aligned} p_{01}^\beta &= \alpha_1\left\{E_{0I}^\beta - Fp_{02}^\beta \exp(\mathrm{i}k_0 R)\right\}, \\ p_{02}^\beta &= \alpha_2\left\{E_{0I}^\beta \exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{R}) - Fp_{01}^\beta \exp(\mathrm{i}k_0 R)\right\}, \\ &\qquad \beta = x, z \end{aligned} \tag{3.6}$$

where

$$G = \frac{1}{R^3} - \mathrm{i}\frac{k_0}{R^2}, \qquad F = G - \frac{k_0^2}{R}. \tag{3.7}$$

The system of algebraic equations (3.5, 3.6) is linear, therefore, it is possible to find its solution by any of standard methods. As a result, we come to the following formulas for unknown quantities:

$$\begin{aligned} p_{01}^y &= \alpha_1\frac{1 + 2\alpha_2 G\exp\left[\mathrm{i}(k_0 R + \mathbf{k}_0\cdot\mathbf{R})\right]}{1 - 4\alpha_1\alpha_2 G^2\exp(\mathrm{i}2k_0 R)}E_{0I}^y, \\ p_{01}^\beta &= \alpha_1\frac{1 - \alpha_2 F\exp\left[\mathrm{i}(k_0 R + \mathbf{k}_0\cdot\mathbf{R})\right]}{1 - \alpha_1\alpha_2 F^2\exp(\mathrm{i}2k_0 R)}E_{0I}^\beta, \\ p_{02}^y &= \alpha_2\frac{\exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{R}) + 2\alpha_1 G\exp(\mathrm{i}k_0 R)}{1 - 4\alpha_1\alpha_2 G^2\exp(\mathrm{i}2k_0 R)}E_{0I}^y, \\ p_{02}^\beta &= \alpha_2\frac{\exp(\mathrm{i}\mathbf{k}_0\cdot\mathbf{R}) - \alpha_1 F\exp(\mathrm{i}k_0 R)}{1 - \alpha_1\alpha_2 F^2\exp(\mathrm{i}2k_0 R)}E_{0I}^\beta. \end{aligned} \tag{3.8}$$

The corresponding expressions for the strengths of fields are easily to find if to take advantage of formula (3.3).

Thus, the solution of self-consistent problem is found in the case when external field is created by the plane wave with frequency ω.

Before to write down the formulas for the field at each atom, we shall make one more simplification. Let the own frequencies of atoms be identical, *i.e.*, $\omega_1 = \omega_2 = \omega_0$. Then, $\alpha_1 = \alpha_2 = \alpha$ and we obtain with formulas (3.8) the expressions for the complex amplitudes of the field

on each of atoms in the following form:

$$E_{01}^{y} = \frac{1 + 2\alpha G \exp\left[\mathrm{i}(k_0 R + \mathbf{k}_0 \cdot \mathbf{R})\right]}{1 - 4\alpha^2 G^2 \exp\left(\mathrm{i}2k_0 R\right)} E_{0I}^{y},$$

$$E_{01}^{\beta} = \frac{1 - \alpha F \exp\left[\mathrm{i}(k_0 R + \mathbf{k}_0 \cdot \mathbf{R})\right]}{1 - \alpha^2 F^2 \exp\left(\mathrm{i}2k_0 R\right)} E_{0I}^{\beta},$$

$$E_{02}^{y} = \frac{\exp\left(\mathrm{i}\mathbf{k}_0 \cdot \mathbf{R}\right) + 2\alpha G \exp\left(\mathrm{i}k_0 R\right)}{1 - 4\alpha^2 G^2 \exp\left(\mathrm{i}2k_0 R\right)} E_{0I}^{y},$$

$$E_{02}^{\beta} = \frac{\exp\left(\mathrm{i}\mathbf{k}_0 \cdot \mathbf{R}\right) - \alpha F \exp\left(\mathrm{i}k_0 R\right)}{1 - \alpha^2 F^2 \exp\left(\mathrm{i}2k_0 R\right)} E_{0I}^{\beta}. \qquad (3.9)$$

Thus, one can see from formulas (3.9) that the field in the location of atoms, generally speaking, does not coincide the external field. Everything is determined by the magnitude of factors αF and αG, which are dependent on the frequency of external field and distance between atoms. Fields (3.9) coincide the external field only in the case when each of the indicated factors much less than 1. The last condition can hold when either distance between atoms is rather large or the frequency of an external field considerably differs from resonant frequency.

Let us relate the complex amplitudes of fields on each atom with each other. It can be made as follows:

$$\mathbf{E}_{02} = \left(E_{01}^{x}\mathbf{x}_0 + E_{01}^{z}\mathbf{z}_0\right)\exp\left(\mathrm{i}k_{\mathrm{r}}R\right) + E_{01}^{y}\exp\left(\mathrm{i}k_{\mathrm{l}}R\right)\mathbf{y}_0, \qquad (3.10)$$

where $\mathbf{x}_0$, $\mathbf{y}_0$, $\mathbf{z}_0$ are the unit vectors of coordinate system,

$$k_{\mathrm{r}} = \frac{\mathbf{k}_0 \cdot \mathbf{R}}{R} - \frac{\mathrm{i}}{R}\ln\frac{1 - \alpha F \exp\left[\mathrm{i}(k_0 R - \mathbf{k}_0 \cdot \mathbf{R})\right]}{1 - \alpha F \exp\left[\mathrm{i}(k_0 R + \mathbf{k}_0 \cdot \mathbf{R})\right]},$$

$$k_{\mathrm{l}} = \frac{\mathbf{k}_0 \cdot \mathbf{R}}{R} - \frac{\mathrm{i}}{R}\ln\frac{1 + 2\alpha G \exp\left[\mathrm{i}(k_0 R - \mathbf{k}_0 \cdot \mathbf{R})\right]}{1 + 2\alpha G \exp\left[\mathrm{i}(k_0 R + \mathbf{k}_0 \cdot \mathbf{R})\right]}. \qquad (3.11)$$

Expression (3.10) in view of formula (2.6) means that the field in the system is the superposition of two waves: transverse wave with wave vector $\mathbf{k}_{\mathrm{r}}$ and longitudinal wave with wave vector $\mathbf{k}_{\mathrm{l}}$. The direction of each mentioned vector coincides the direction of y-axis (Fig. 1).

Take up the point about the reasons of origin of longitudinal wave. It follows from formula (3.10) that longitudinal wave takes place always when y-component of complex amplitude of field at the first atom is nonzero. Now, using formulas (3.9), it is easy to make a conclusion that the indicated condition is held if y-component of external field is not equal to zero. Thus, if, for example, the wave vector of incident wave $\mathbf{k}_0$ is parallel to y-axis, only transverse wave propagates in the system. It is necessary to notice that the situation when only longitudinal wave propagates in the system is impossible, since, in this case E_{0I}^{x}, E_{0I}^{z} are equal to zero.

4 Laws of dispersion for longitudinal and transverse waves inside small object

Let us investigate the dependence of k_{r} and k_{l} upon frequency (that is the same, upon modulus of vacuum wave vector k_0). At first, we consider the dependence of k_{r} on k_0. Formula (3.11) yields the behaviour of real part k'_{r} and imaginary part k''_{r} of wave vector $\mathbf{k}_{\mathrm{r}}$. Both relations are presented in Figure 2.

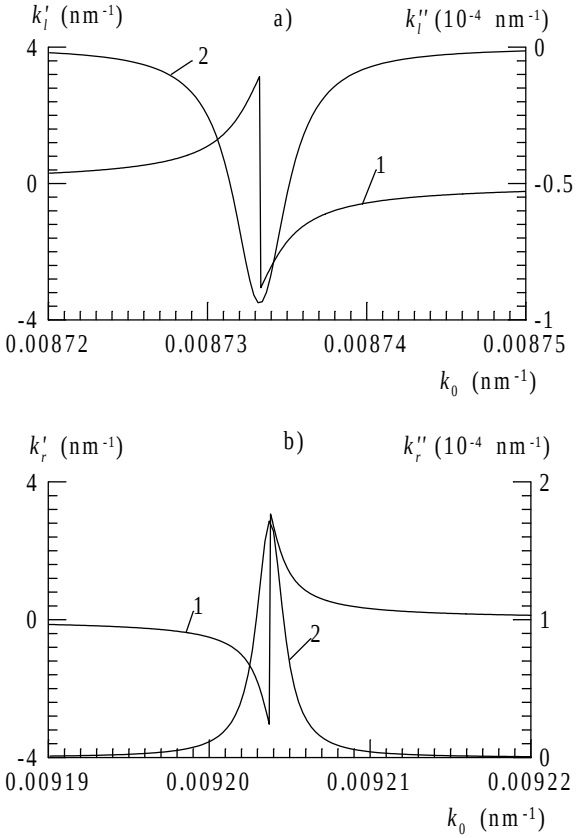

Fig. 2. Laws of dispersion for longitudinal (a) and transverse (b) waves. Curve 1 represents k'_{r}, k'_{l}, curve 2 represents k''_{r}, k''_{l}. The angle between $\mathbf{k}_0$ and $\mathbf{R}$ equals to 45°, $R = 1$ nm, $\omega_0 = 2.715 \times 10^{15}$ s^{-1}, $T = 10^{-8}$ s.

One can see from Figure 2b that k'_{r} considerably varies when $0.00919 < k_0 < 0.00922$ nm^{-1}. As k'_{r} decreases with increase of k_0, it is possible to call such a dispersion negative. The value of k''_{r} is also strongly varies in this area. The sign of k''_{r} is positive that corresponds to absorption of transverse wave.

The dependence of k_{l} upon k_0 is similar to the relations just examined. However, there are some important features. First, the dispersion of k'_{l} is positive. Second, k''_{l} has negative sign that corresponds to the amplification of longitudinal wave.

The dependence of dispersion law upon distance between atoms is also of interest. This distance being increased, the areas where the essential changes of k_{r} and k_{l} take place come nearer to each other and their values come nearer to vacuum values.

Now, we take up the point about change of dispersion law for k''_{r}, k''_{l} with the change of angle between axis of system, *i.e.* y-axis, and wave vector of incident wave. This angle being increased from 0° up to 90°, the decreasing of the width of resonance characteristics and simultaneous increase of the maximum of the modulus of appropriate

quantities are observed. The changes can be rather significant. So, if the angle is approximately equal to zero the maximum of $|k_1''|$ is equal to 0.00008 nm^{-1}, and if the angle will be increased up to 85°, the corresponding maximum is increased up to 0.0003 nm^{-1}.

5 Electric field of light wave outside the small object

Let us substitute expressions (3.8) into equation (2.1) and obtain the field of light wave in the observation points outside the small object. In this case Green function $\exp(\mathrm{i}k_0R_j)/R_j$ relates induced dipole j and observation point $\mathbf{r}$ outside the small object.

Let us consider the field of light wave in wave zone relative to the origin of coordinates. Let us assume that the size of object is significantly less than wavelength of light. Hence, the approximate equality $R_j \approx r$ is held where r is the distance from the origin of coordinates to the observation point. The external wave is supposed to be linearly polarized in the plane containing both atoms, *i.e.* in the plane zy.

In wave zone, where $r \gg k_0^{-1}$, the principal role is played by retarding terms of expression (2.7) inversely proportional to relative distance r. One can be convinced that the remaining terms in (2.7) yield the considerably small contribution. Therefore, we have the following formula for the electric field strength of light wave in wave zone:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_I(\mathbf{r},t) + \frac{k_0^2}{r}\mathbf{n}\times(\mathbf{p}\times\mathbf{n})\exp(\mathrm{i}k_0R) + \mathrm{cc}, \quad (5.1)$$

where $\mathbf{n} = \mathbf{r}/r$ is the unit vector directed from the small object to observation point, $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$ is the resultant dipole moment of object. Vector $\mathbf{p}$ is easily to find using the formulas (3.8). Indeed, taking into account that the quantities $\mathbf{p}_1$ and $\mathbf{p}_2$ are harmonic functions of time, we come to the following formula for $\mathbf{p}$:

$$\mathbf{p} = \mathbf{p}_0\exp(-\mathrm{i}\omega t) + \mathrm{cc}, \quad (5.2)$$

where

$$p_0^y = \alpha\frac{1+2\alpha G\exp(\mathrm{i}k_0R)}{1-4\alpha^2G^2\exp(\mathrm{i}2k_0R)}\left(1+\exp[\mathrm{i}\mathbf{k}_0\cdot\mathbf{R}]\right)E_{0I}^y,$$
$$p_0^\beta = \alpha\frac{1-\alpha F\exp(\mathrm{i}k_0R)}{1-\alpha^2F^2\exp(\mathrm{i}2k_0R)}\left(1+\exp[\mathrm{i}\mathbf{k}_0\cdot\mathbf{R}]\right)E_{0I}^\beta. \quad (5.3)$$

The end of vector $\mathbf{p}$ in general case describes an ellipse in space, therefore, we can represent vector $\mathbf{p}_0$, defined by formulas (5.3), as:

$$\mathbf{p}_0 = (\mathbf{p}_0' + \mathrm{i}\mathbf{p}_0'')\exp(-\mathrm{i}\delta), \quad (5.4)$$

where $\mathbf{p}_0'$, $\mathbf{p}_0''$ are two mutually perpendicular material vectors, δ is some real number.

The substitution of expression (5.4) into the formula (5.1) results in the decomposition of the field in wave zone into two linearly polarized waves which planes of polarization and amplitudes are determined by vectors $\mathbf{p}_0'$ and $\mathbf{p}_0''$. Numerical analysis shows that the magnitude of $|\mathbf{p}_0''|/|\mathbf{p}_0'|$ is much less than 1 in all optical range, therefore, it is possible to regard with large accuracy that the field in question is completely determined by vector $\mathbf{p}_0'$. The latter means that the field in wave zone is linearly polarized. However, unlike the case when the field in wave zone is created by one dipole, the orientation of vector $\mathbf{p}_0'$ depends on the frequency of external field. One can see from Figure 3 that the angle between vector determining polarization of external wave and vector $\mathbf{p}_0'$ is essentially varies in some range of k_0.

Thus, relative to the field in wave zone, the pair of atoms behaves like a dipole which axis can change its position in space with changing the frequency of external field.

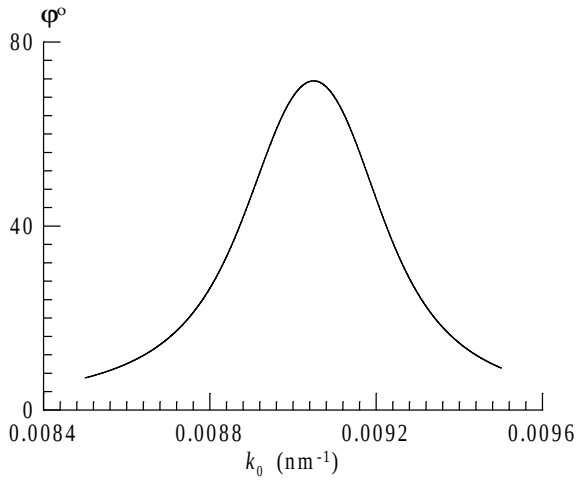

Fig. 3. Angle between vector $\mathbf{p}_0'$ and vector of polarization of external wave φ as a function of frequency. The angle between $\mathbf{k}_0$ and $\mathbf{R}$ equals to 45°, $R = 1$ nm, $\omega_0 = 2.715\times10^{15}$ s^{-1}, $T = 10^{-8}$ s. Corresponding curve for one atom concurs with k_0 axis.

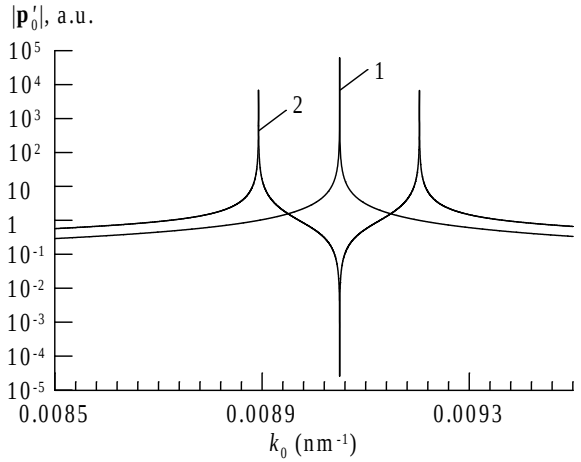

Fig. 4. Amplitude of field in wave zone as a function of wave number: curve 1 corresponds to single atom, curve 2 corresponds to two atoms.

Now, let's take up the point about the amplitude characteristics of the field. The analysis of the dependence of vector $\mathbf{p}'_0$ modulus upon k_0 shows that it always has two maxima situated at different sides of the maximum that would appear in the case of one atom. Difference between values of k_0 corresponding to these maxima depends on interatomic distance. The latter being increased, this difference decreases and the magnitude of maxima increases (Fig. 4).

Conclusion

Thus, in this paper the interaction of small object formed by two atoms is investigated with an external field. We obtained the expressions for electric fields inside and outside of small object. The investigation of electric field in wave zone has demonstrated that the presence of the second atom in the system influences not only the amplitude characteristics of the field, but also results in the change of the appropriate polarizing characteristics. It means that it is possible to obtain the information about intrinsic properties of small object formed by two dipole atoms with the aid of specially organized experiment.

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